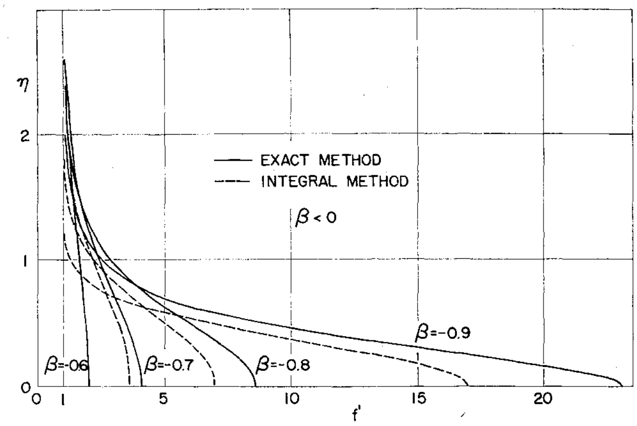

Fig. 2 Velocity profiles for $-1.0 < \beta < -0.5$.

$\neq 0$. However, the factor $u_e^2/2h_e$ is generally a function of s and therefore prevents rigorous similarity of G. Locally similar approximate solutions also may be useful in this case.

The remainder of this note is concerned with flows wherein $u_e^2/2h_e[1 - (F'/\eta)^2] \ll 1.0$. With this condition, $J = \eta$ and Eq. (8a) reduces to Eq. (1), with $\beta = (2s/u_e)(du_e/ds)(H_e/h_e)$. Solutions of Eqs. (1) have been obtained by numerical integration (utilizing an IBM 7040 digital computer) and by a momentum integral method. The computer program is analogous to the one discussed in Ref. 4.

Direct integration of (1a), with (1b) and $(F'/\eta)_\infty' = 0$, yields

$$\int_0^\infty \frac{F'}{\eta}\left(1 - \frac{F'}{\eta}\right)\eta d\eta + \beta\int_0^\infty \left[1 - \left(\frac{F'}{\eta}\right)^2\right]\eta d\eta = 0 \tag{11}$$

Approximate solutions are derived by assuming

$$F'/\eta = 1 + [(F'/\eta)_0 - 1]e^{-b\eta^2/4} \tag{12}$$

where $(F'/\eta)_0$ and b are constants. $(F'/\eta)_0$ is determined by satisfying Eq. (11), and b by evaluating Eq. (1a) at $\eta = 0$. There results

$$(F'/\eta)_0 = -[(1 + 3\beta)/(1 + \beta)] \qquad b = 2\beta^2/(1 + \beta)$$

Furthermore, the following exact result can be derived from Eq. (11): $\delta^*/\theta = -[(1 + \beta)/\beta]$; where δ^* and θ are, respectively, the displacement and momentum thicknesses.

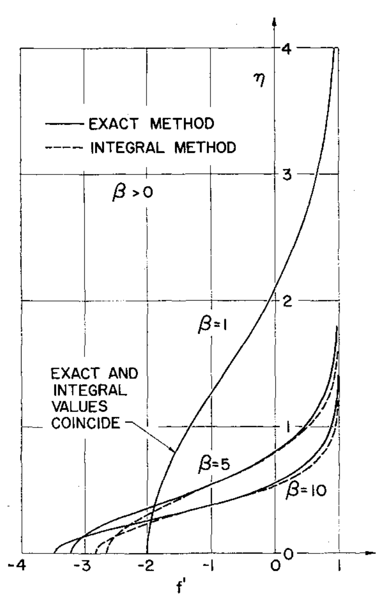

Fig. 3 Velocity profiles for $\beta > 0.5$.

The pertinent results are presented in Table 1 and Figs. 1–3. The exact results in the range $-0.5 < \beta < 0$ have been taken from Ref. 3. The new results presented here show that for $-1.0 < \beta < -0.5$ the F'/η profiles are monotonic and jet-like, whereas in the range $0.5 < \beta < \infty$ the F'/η profiles are monotonic and wake-like. The possibility of deriving solutions when $0 < \beta < 0.5$ is now being investigated.

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Wave Intersection Limits for Focused Compression of Air in Chemical Equilibrium

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INLETS for hypersonic ramjets[1, 2] using isentropic, external compression have attractive weight-saving features. These inlets, at their design points, require that the compression-surface-generated, one-family characteristics be focused near the cowl leading edge to minimize spillage drag. The first design consideration should be the flow-turning limitations imposed by the shock structure at the focal point. It is the purpose of this paper to present these limitations as calculated for equilibrium air to Mach number 8 for two-dimensional flow. A comparison of these results with an extension of the work by Connors and Meyer[3] for a perfect gas is also presented.

As stated in Ref. 3, the limiting amount of isentropic flow turning with focused characteristics is determined from analysis of the branch-shock configuration, which consists of a single intersection of an isentropic compression fan, a reflected wave, a vortex sheet, and shock wave (Fig. 1). Theoretical requirements of any wave intersection are that equal static pressures and flow direction exist on either side of the

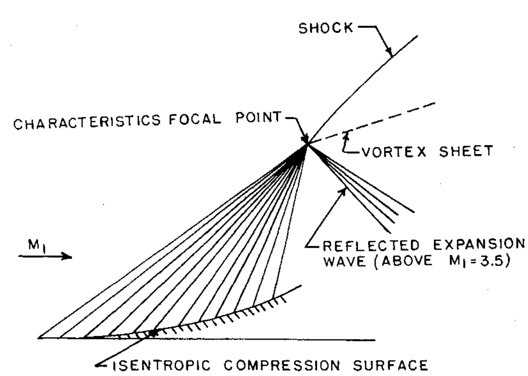

Fig. 1 Schematic of the branch-shock structure analyzed for maximum isentropic compressive turning.

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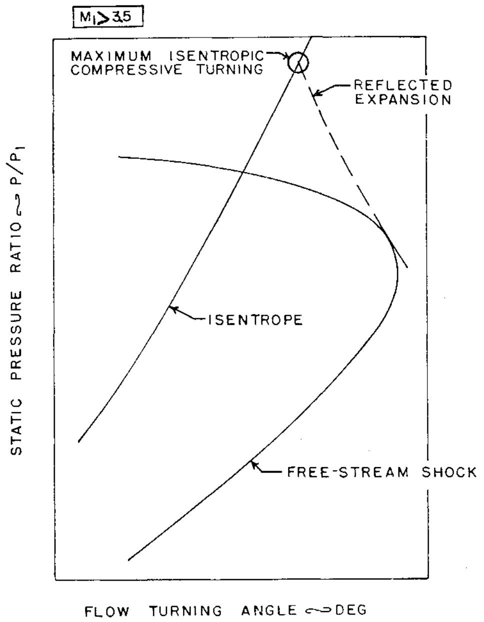

Fig. 2 Typical pressure-deflection polar.

vortex sheet. In order to satisfy these requirements, the theoretical solution requires an intersection of the reflected-wave polar with the freestream-shock polar. The limiting condition occurs at a point on the isentrope corresponding to the maximum deflection angle from which a reflected-wave polar will be just tangent to the freestream-shock polar (Fig. 2).

The foregoing analysis was carried out in the Mach 4–8 range for equilibrium air assuming constant dynamic pressure trajectories of 500 and 2000 psf. The shock polars and isentropes were calculated by a digital computer using air properties programmed from Ref. 4. The limiting amount of compression was determined graphically from these data. The results are compared in Fig. 3 with extension of the perfect gas data of Ref. 3. It is seen that little difference in isentropic turning limits exists for the two trajectories studied. This suggests that the limits shown can be used to engineering accuracy anywhere in the included trajectory envelope. A marked deviation exists above Mach 3.5, however, between the isentropic turning limits for air and perfect gas. Since the pressure-deflection isentrope is nearly identical for

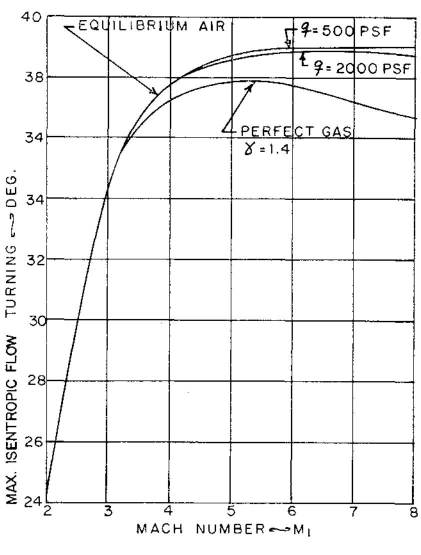

Fig. 3 Flow turning limit for two-dimensional, focused, isentropic compression.

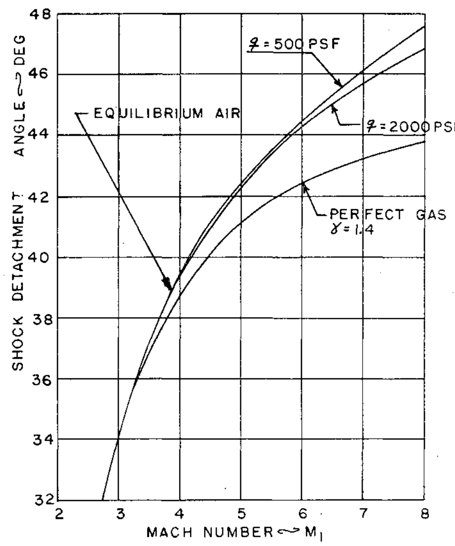

Fig. 4 Shock detachment angle vs freestream Mach number.

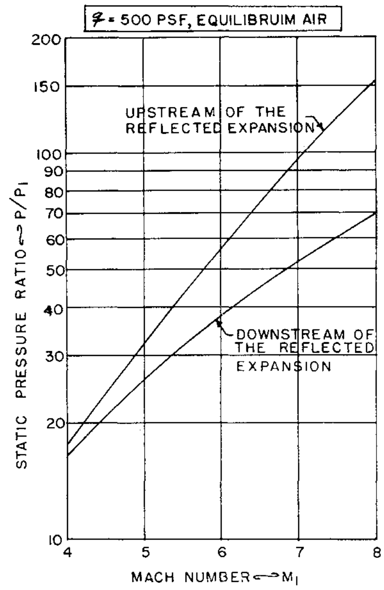

Fig. 5 Pressure ratio upstream and downstream of the reflected expansion for maximum isentropic compression.

perfect gas and real air in this range, the difference in turning limits is due primarily to differences in the freestream-shock polar. The detachment angle for a freestream shock, presented in Fig. 4, demonstrates this effect.

A final point of interest at the higher Mach numbers is the large difference in pressure across the reflected expansion (Fig. 1). The pressure before and after the reflected expansion is shown in Fig. 5 for the Mach 4–8 range. This indicates the benefit to inlet performance of properly locating the cowl to prevent this reflected expansion from entering the inlet.

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